AP Calculus AB/BC #1 Key

A) $\int_{60}^{135} f(t)\,dt$ The amount of gas in gallons pumped into the gas tank between $t=60$ seconds and $t=135$ seconds after pumping began.

RHS $\int_{60}^{135} f(t)\,dt \approx 30(0.15)+30(0.1)+15(0.05)=8.25$ gallons

B) Yes, $f(t)$ is differentiable and therefore continuous on $[60,120]$. Also, $f'(c)=\dfrac{f(120)-f(60)}{120-60}=\dfrac{0}{60}=0$. Therefore,

by MVT, there is at least one value of c for $60<c<120$ where $f'(c)=0$.

C) $\frac{1}{150}\int_{0}^{150} g(t)\,dt = 0.0959$ gallons/second

D) $g'(140)=-0.0049$

The rate of flow of gasoline is decreasing at a rate of $0.0049 \frac{\text{gallons}}{\text{second}}$ per second when $t=140$ seconds

AP Calculus AB #2 Key

A) Stephen changes direction once on the interval $0 < t < 90$ seconds. At $t = 56$ $v(56) = 0$ and $v(t)$ changes signs from positive to negative here.

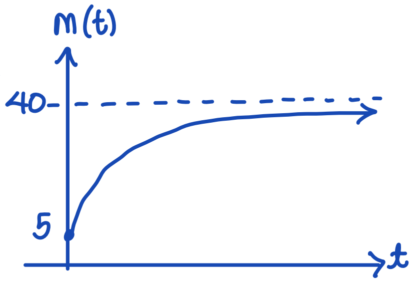
B) $a(60) = v'(60) = -0.0360$ m/s per second
$v(60) = -0.159$ or -0.160
At $t = 60$, Stephen is speeding up because $v(60) < 0$ and decreasing [velocity and acceleration have the same sign.]

C) $\int_{20}^{80} v(t)\,dt = s(80) - s(20) = 23.3839$ meters

D) Total distance on $[0, 90]$ $= \int_0^{90} |v(t)|\,dt = 62.1642$ meters

AP Calculus AB/BC #3 Key

A)



B) $\frac{dM}{dt} = \frac{1}{4}(40 - M(0)) = \frac{1}{4}(40-5) = \frac{35}{4}$

Tangent line: $y = \frac{35}{4}t + 5$

$M(2) = \frac{35}{4}(2) + 5$

$M(2) = \underline{\underline{\frac{45}{2}{}^\circ C}}$

C) $\frac{dM}{dt} = \frac{1}{4}(40-M)$

$\frac{d^2M}{dt^2} = \frac{1}{4}\left(-\frac{dM}{dt}\right)$

$= -\frac{1}{4}\left[\frac{1}{4}(40-M)\right]$

$= -\frac{1}{16}(40-M)$

$\frac{d^2M}{dt^2} < 0$ for all t and $M(t)$ is increasing, so the tangent line approximation for part B will overestimate $M(2)$ since the graph of $M(t)$ is concave down and tangent line lies above the curve on $[0,2]$

D) $\frac{dM}{dt} = \frac{1}{4}(40-M)$ $\quad M(0) = 5$

$\int \frac{dM}{40-M} = \int \frac{1}{4}\,dt$

$-\ln|40-M| = \frac{1}{4}t + C$

$-\ln|35| = \frac{1}{4}(0) + C$

$C = -\ln(35)$

$-\ln|40-M| = \frac{1}{4}t - \ln(35)$

$\ln|40-M| = -\frac{1}{4}t + \ln(35)$

$e \qquad e$

$40 - M = 35e^{-1/4t}$

$\underline{\underline{M = 40 - 35e^{-1/4t}}}$

note: $M(0) = 5$

AP Calculus AB/BC #4 Key

A) f has no relative extrema at $x=6$ since $f'(5)>0$ and $f'(6)=0$ but f' does not change signs at $x=6$. f is increasing on the interval $2<x<8$.

B) On $(-2,0)$ and $(4,6)$ since f is concave down and f' is decreasing on $(-2,0)$ and on $(4,6)$ f' is decreasing $f''<0$.

C) $\lim\limits_{x\to 2} \dfrac{6f(x)-3x}{x^2-5x+6}$ $\quad \lim\limits_{x\to 2} 6f(x)-3x = 0$ and $\lim\limits_{x\to 2} x^2-5x+6 = 0$

$\therefore$ By L'Hospitals Rule

$$\lim_{x\to 2} \frac{6f'(x)-3}{2x-5} = \frac{6f'(2)-3}{2(2)-5} = \frac{-3}{-1} = \underline{\underline{3}}$$

D) Abs. minimum occurs at endpoint of relative minimum. f has a rel. min at $x=2$ since f' changes sign from negative to positive at $x=2$. By candidate's test:

x	$f(x)$
-2	$1+\frac{1}{2}(3)(2)-\frac{1}{2}(2)(1)=3$
* 2	1
8	$1+\int_2^8 f'(x)\,dx = 11-2\pi$

Note: on $2<x<8$ $f'>0$ so $\int_2^8 f'(x)\,dx>1$

$f(8)>f(2)$ $\therefore$ by candidate's test, f has an absolute minimum value of 1

AP Calculus AB #5 Key

A) $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(7) = f'(g(7)) \cdot g'(7)$

$h'(7) = f'(0) \cdot 8$

$h'(7) = \frac{3}{2}(8) = \underline{\underline{12}}$

B) $K'(x) = [f(x)]^2 \cdot g(x)$

$K''(x) = 2[f(x)] \cdot f'(x) \cdot g(x) + [f(x)]^2 \cdot g'(x)$

$K''(4) = 2(4)(3)(-3) + (4)^2 \cdot 2$

$K''(4) = -72 + 32$

$\therefore$ The graph of K is concave down at $x = 4$ because $K''(4) < 0$.

C) $m(2) = 5(2)^3 + \int_0^2 f'(t)\,dt$

$= 40 + [f(2) - f(0)]$

$= 40 + [7 - 10]$

$= \underline{\underline{37}}$

D) $m'(x) = 15x^2 + f'(x)$

$m'(2) = 15(2)^2 + f'(2)$

$= 60 - 8$

$= \underline{\underline{52}}$

$\therefore$ m is increasing at $x = 2$ since $m'(2) > 0$.

AP Calculus AB #6 Key

A) $6xy = 2 + y^3$

$6y + 6xy' = 3y^2y'$

$\frac{dy}{dx}\left[3y^2 - 6x\right] = 6y$

$\frac{dy}{dx} = \frac{\cancel{3}(2y)}{\cancel{3}(y^2-2x)} = \frac{2y}{y^2-2x}$

B) Tangent line is horizontal when $\frac{dy}{dx} = 0$ and $2y = 0$ when $y = 0$

But $6x(0) = 2 + (0)^3$ and $0 \neq 2$ so, not possible.

There is no point on the curve where the tangent line is horizontal since there is no solution for $(x, 0)$ where $\frac{dy}{dx} = 0$.

C) Tangent line is vertical when $\frac{dy}{dx}$ is undefined, so

$y^2 - 2x = 0$

$x = \frac{y^2}{2}$

$x = 1/2$

$\Rightarrow$

$6\left(\frac{y^2}{2}\right)y = 2 + y^3$

$3y^3 = 2 + y^3$

$y^3 = 1$

$y = 1$

$\left(\frac{1}{2}, 1\right)$

D) $6x(t)\,y(t) = 2 + [y(t)]^3$

$6x(t)\,y'(t) + 6x'(t)\,y(t) = 3[y(t)]^2 \cdot y'(t)$

$6\left(\frac{1}{2}\right)y'(t) + 6(-2)\left(\frac{2}{3}\right) = 3(-2)^2 \cdot y'(t)$

$3y'(t) - 8 = 12y'(t)$

$9y'(t) = -8$

$y'(t) = \frac{-8}{9}\ \frac{\text{units}}{\text{second}}$

AP Calculus BC #2 Key

$x'(t) = e^{\cos t}$
$x''(t) = -\sin x \cdot e^{\cos x}$

$y(t) = 2\sin t$
$y'(t) = 2\cos t$
$y''(t) = -2\sin t$

A) $a(t) = \langle x''(t), y''(t) \rangle$
$a(1) = \langle -\sin(1) e^{(\cos 1)}, -2\sin(1) \rangle$
$a(1) = \langle -1.444, -1.6829 \rangle$

B) speed $= \sqrt{[x'(t)]^2 + [y'(t)]^2} = 1.5$
$\underline{t = 1.254}$

C) $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$
$\left.\frac{dy}{dx}\right|_{t=1} = \frac{2\cos 1}{e^{\cos 1}} = 0.6295$

$x(1) = x(0) + \int_0^1 x'(t)\,dt$
$= 1 + \int_0^1 x'(t)\,dt$

$\underline{\underline{x(1) = 3.3415}}$

D) $\int_0^{\pi} \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt = 6.0346$

AP Calculus BC #5 Key

A) $g(3)=2$

$$\int_0^3 [f(x)-g(x)]dx = \int_0^3 f(x)dx - \int_0^3 \frac{12}{3+x}dx$$

$$= 10 - \left(12\ln|3+x|\right)\Big]_0^3$$

$$= 10-(12\ln 6 - 12\ln 3)$$

$$= \underline{\underline{10-12\ln 2}}$$

B) $$\lim_{b\to\infty}\int_0^b [g(x)]^2 dx = \lim_{b\to\infty}\int_0^b \left(\frac{144}{(3+x)^2}\right)dx$$

$$= \lim_{b\to\infty}\left[\frac{-144}{3+x}\Big|_0^b\right] = \lim_{b\to\infty}\left[\frac{-144}{3+b} - \frac{-144}{3}\right]$$

$$= \underline{\underline{\frac{144}{3}}}$$

C) $h(x) = x\cdot f'(x)$

$\int uv'dx = uv - \int u'v\,dx$

$$\int_0^3 x\cdot f'(x)dx = x\cdot f(x)\Big|_0^3 - \int_0^3 f(x)\,dx$$

$$= 3f(3) - 0 - 10$$

$$= 3(2)-10 = \underline{\underline{-4}}$$

AP Calculus BC # 6 Key

A) $f^{(4)}(x) = -2x \cdot 2x \cdot f''(x^2) + (-2) \cdot f'(x^2)$

$f''(0) = -f(0) = -2$

$f'''(0) = -2(0) \cdot f'(0) = 0$

$f^{(4)}(0) = f'(0)(-2) = 3(-2) = -6$

$$T_4(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2} + \frac{f'''(0)x^3}{3!} + \frac{f^{(4)}x^4}{4!}$$

$$T_4(x) = 2 + 3x + \left(\frac{-2}{2}\right)x^2 + 0x^3 + \left(\frac{-6}{24}\right)x^4$$

$$\underline{\underline{T_4(x) = 2 + 3x - x^2 - \frac{1}{4}x^4}}$$

B) $$|f(0.1) - T_4(0.1)| < \left| \frac{f^{(5)}(x)}{5!} x^5 \right|$$

$$\leq \frac{15}{5!}(0.1)^5 = \frac{15}{5!}\left(\frac{1}{10}\right)^5 = \frac{15}{5!}\left(\frac{1}{10^5}\right)$$

$\therefore$ since $\frac{15}{5!} < 1$, $|f(0.1) - T_4(0.1)| < \frac{1}{10^5}$

C) $g'(0) = e^0 f(0) = 1(2) = 2$

$g''(0) = e^0 f'(0) + f(0) \cdot e^0$

$= 3 + 2$

$= 5$

$$\underline{\underline{T_2(x) = 4 + 2x + \frac{5}{2}x^2}}$$