AP Calculus AB/BC #1 Key

2024

A) $C'(5) = \dfrac{C(7) - C(3)}{7-3} = \dfrac{69-85}{7-3} = -4$

$C'(5) = -4\ C°/\text{min}$

B) $\int_0^{12} C(t)\,dt = (3-0)(100) + (7-3)(85) + (12-7)(69)$

$= 3(100) + 4(85) + 5(69)$

$= 300 + 340 + 345$

$= 985$

$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of coffee in degrees Celsius as it cools in the cup over time $0 \le t \le 12$ min.

C) $\int_{12}^{20} C'(t)\,dt = C(20) - C(12)$

$-14.670811944 = C(20) - 55$

$C(20) \approx 40.32918\ C°$

D) $C''(t) > 0$ on $12 < t < 20$, so temperature of coffee is changing at an increasing rate on the interval.

AP Calculus BC #2 Key

A) $\sqrt{(x'(2))^2 + (y'(2))^2} = 12.3048506$

At time $t=2$, the speed of the particle is 12.305 cm/sec.

B) $\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 15.901715$

The total distance traveled by the particle over the time interval $0 \le t \le 2$ is 15.902 cm.

C) $\int_0^2 y'(t)\,dt = y(2) - y(0)$ @ $t=2$; $(x,y)=(3,6)$

$7.173613 = 6 - y(0)$

$y(0) = -1.173613$

D) $y(t) > 0$ where is $y'(t) < 0$ QI (x,y) positive

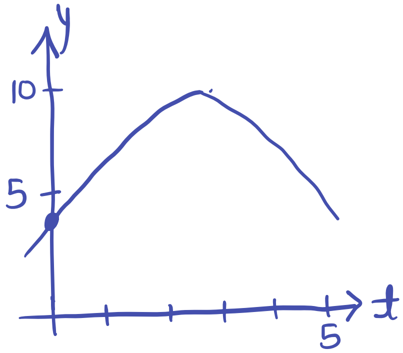
@ $t \approx 5.22183$ $y'(t) = 0$

On interval from $2 \le t \le 8$ $y'(t) = 0$ at $t = 5.222$ seconds

$\therefore$ $y(t) > 0$ and $y'(t) < 0$ on $(5.222, 8]$

AP Calculus AB/BC #3 Key

A)



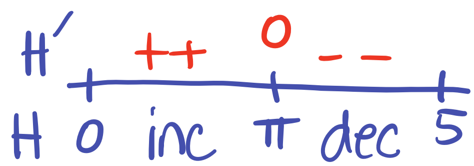
$H(0) = 4$

B) $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$0 = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$\cos\left(\frac{t}{2}\right) = 0$

$t = \pi$



At $t=\pi$ $H'(\pi)=0$ and $H(t)$ changes from increasing to decreasing at $t=\pi$. This critical point is a relative maximum for depth of seawater.

c) $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$\int \frac{2}{H-1}\, dH = \int \cos\left(\frac{t}{2}\right) dt$

$2 \ln|H-1| = 2\sin\left(\frac{t}{2}\right) + C$

$\ln|H-1| = \sin\left(\frac{t}{2}\right) + C$

$H(0) = 4$

$\ln|4-1| = \sin 0 + C$

$\ln 3 = C$

$\ln|H-1| = \sin\left(\frac{t}{2}\right) + \ln 3$

$e \qquad e$

$H - 1 = e^{\sin t/2} \cdot e^{\ln 3}$

$$\underline{\underline{H(t) = 3e^{\sin(t/2)} + 1}}$$

AP Calculus AB/BC #4 Key

A) $g(-6) = \int_0^{-6} f(t)\,dt = -\int_{-6}^{0} f(t)\,dt = \underline{\underline{-12}}$

$g(4) = \int_0^4 f(t)\,dt = \frac{1}{2}(4)(2) = \underline{\underline{4}}$

$g(6) = \int_0^4 f(t)\,dt + \int_4^6 f(t)\,dt = 4 + (-1) = \underline{\underline{3}}$

$\underline{\underline{g(-6) = -12;\ g(4) = 4;\ g(6) = 3}}$

B) $g(x)$ has a critical point where $f(x) = g'(x) = 0$. This occurs $\underline{\underline{\text{at } x = 4}}$ $\quad f(4) = g'(4) = 0$

c) $h(x) = \int_{-6}^{x} f'(t)\,dt$

$h(6) = \int_{-6}^{6} f'(t)\,dt = f(6) - f(-6)$

$= -1 - 0.5$

$\underline{h(6) = -1.5}$

$h'(x) = \frac{d}{dx}\left[\int_{-6}^{x} f'(t)\,dt\right] = f'(x)$

$h'(6) = f'(6) = \underline{\underline{-\frac{1}{2}}}$

$h''(6) = f''(6) = \underline{\underline{0}}$

AP Calculus BC #5 Key

A) $h'(\pi) = \frac{d}{dt}\int_0^x \sqrt{1+(f'(t))^2}\,dt$

$h'(\pi) = \sqrt{1+(f'(\pi))^2}$

$= \sqrt{1+6^2}$

$h'(\pi) = \sqrt{37}$

B) For the differentiable function f(x) on interval $0 \le x \le \pi$, the length of the curve of $y=f(x)$ from 0 to π.

C) $f(\pi) = f(0) + \pi \frac{dy}{dx}_{(0,0)} \Rightarrow 0 + 5\pi = 5\pi$

$f(2\pi) = f(\pi) + \pi \frac{dy}{dx}_{(\pi, 5\pi)}$

$= 5\pi + 6\pi$

$f(2\pi) = 11\pi$

D) $\int (t+5)\cos\left(\frac{t}{4}\right)dt$

$u = t+5 \leftrightarrow v = 4\sin(t/4)$

$du = 1dt \quad dv = \cos(t/4)$

$(t+5)\left[4\sin\left(\frac{t}{4}\right)\right] - \int 4\sin\left(\frac{t}{4}\right)dt$

$(t+5)(4\sin(t/4)) + 16\cos\left(\frac{t}{4}\right) + C$

AP Calculus BC #6 Key

A) $x=6;\ \sum_{n=1}^{\infty} \frac{(n+1)\cancel{6^n}}{n^2\cdot \cancel{6^n}} \Rightarrow \sum_{n=1}^{\infty} \frac{n+1}{n^2}$ Limit Comparison Test

$\frac{n}{n^2} = \frac{1}{n}$ L.C.T. $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test

$$\lim_{n\to\infty} \frac{\frac{n+1}{n^2}}{\frac{1}{\cancel{n}}} \cdot \frac{n}{1} \Rightarrow \lim_{n\to\infty} \frac{n+1}{n} = 1$$

Since $0<1<\infty$ both series diverge.

B) $|f(-3) - S_3| < \frac{1}{50}$

$$|a_{4}| = \left| \frac{4+1}{4^2}\left(\frac{-1}{2}\right)^4 \right| = \frac{5}{16}\cdot\frac{1}{16} \Rightarrow \frac{5}{256}$$

$\therefore \underline{\underline{\frac{5}{256} < \frac{1}{50}}}$

C) $$f'(x) = \sum_{n=1}^{\infty} \frac{(n+1)\cancel{n}x^{n-1}}{n^{\cancel{2}}\cdot 6^n} \Rightarrow \underline{\underline{\sum_{n=1}^{\infty} \frac{(n+1)x^{n-1}}{n\cdot 6^n}}}$$

D) By Ratio Test to check R.O.C.

$$\lim_{n\to\infty} \left| \frac{(n+2)\cancel{x^n}}{(n+1)\,\cancel{6^{n+1}}} \cdot \frac{\cancel{6^n}\cdot n}{(n+1)\,\cancel{x^{n-1}}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{(n+2)}\,n\cdot x}{\cancel{(n+1)}(n+1)\cdot 6} \right| \Rightarrow \left|\frac{x}{6}\right| < 1 \quad \underline{\underline{|x|<6}}$$

FRQ #6 (cont)

D) $$\lim_{n\to\infty}\left|\frac{(n+2)\,x^{2(n+1)}}{(n+1)^2\,3^{n+1}}\cdot\frac{n^2\,3^n}{(n+1)\,x^{2n}}\right|$$

$$\lim_{n\to\infty}\left|\frac{\cancel{(n+2)}\,\cancel{n^2}\,x^2}{\cancel{(n+1)}^3\cdot 3}\right| \Rightarrow \lim_{n\to\infty}\left|\frac{x^2}{3}\right|$$

$$\left|\frac{x^2}{3}\right| < 1$$

$|x| < \sqrt{3}$ radius $= \sqrt{3}$